

Orbit and gravity field: common versus sequential analysis

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*Gravity field mapping methodology from GRACE
and future gravity missions*

Hotine Marussi 2013, Roma

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- Gravity field and Orbit
- Signal and Noise in monthly models (GRACE)
- Timewise analysis: the concept of Lumped Coefficients
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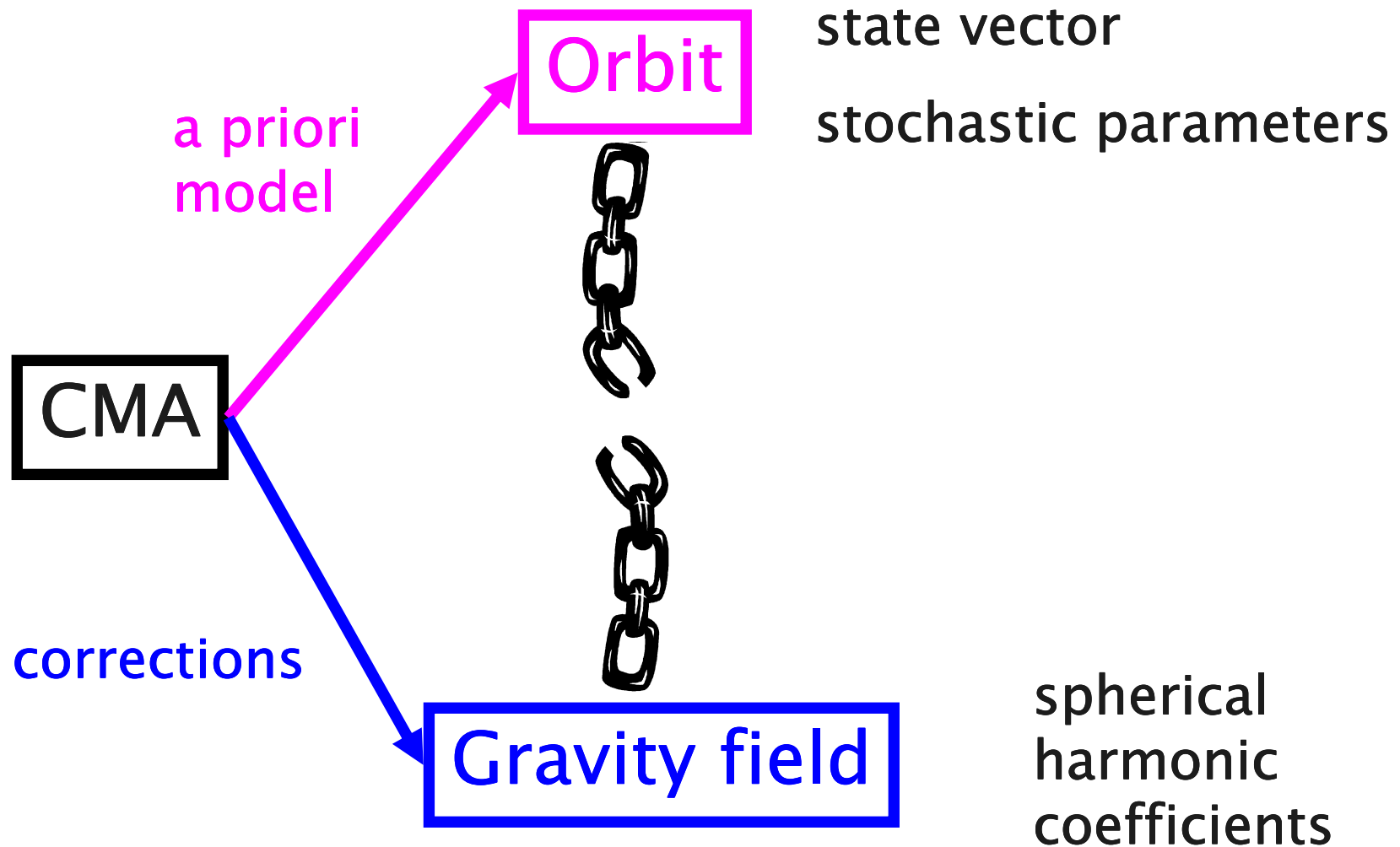
Gravity field and Orbit



Non-linear parameter
estimation problem

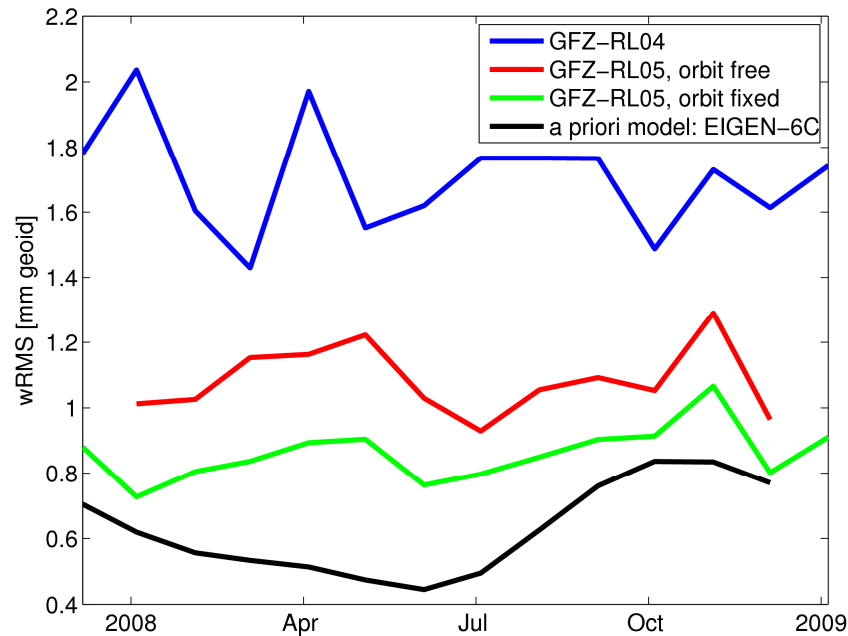
- A priori model (linearization)
- Observations
- Regularization (a priori knowledge via pseudo-observations)

A generalized orbit determination problem



Signal and Noise in monthly models (GRACE)

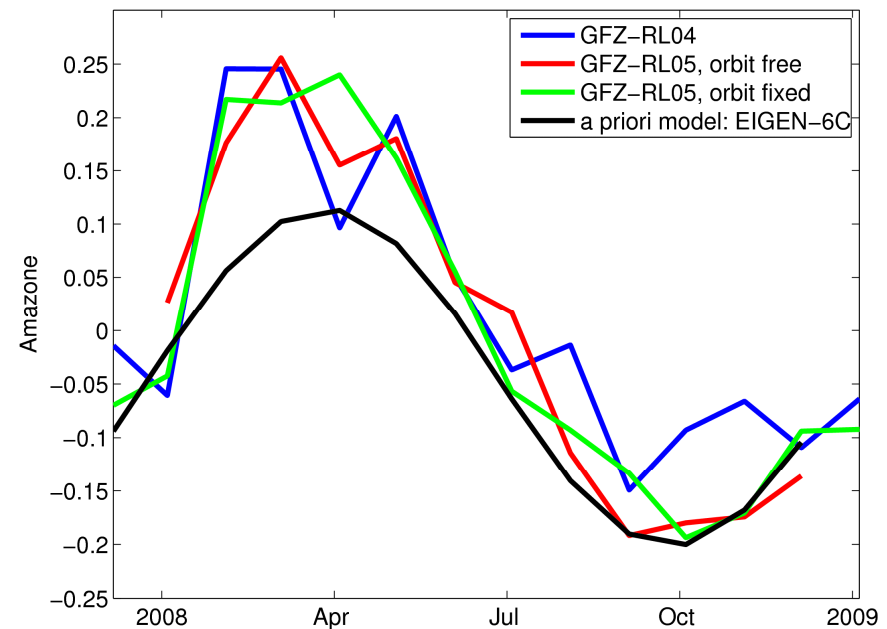
Noise



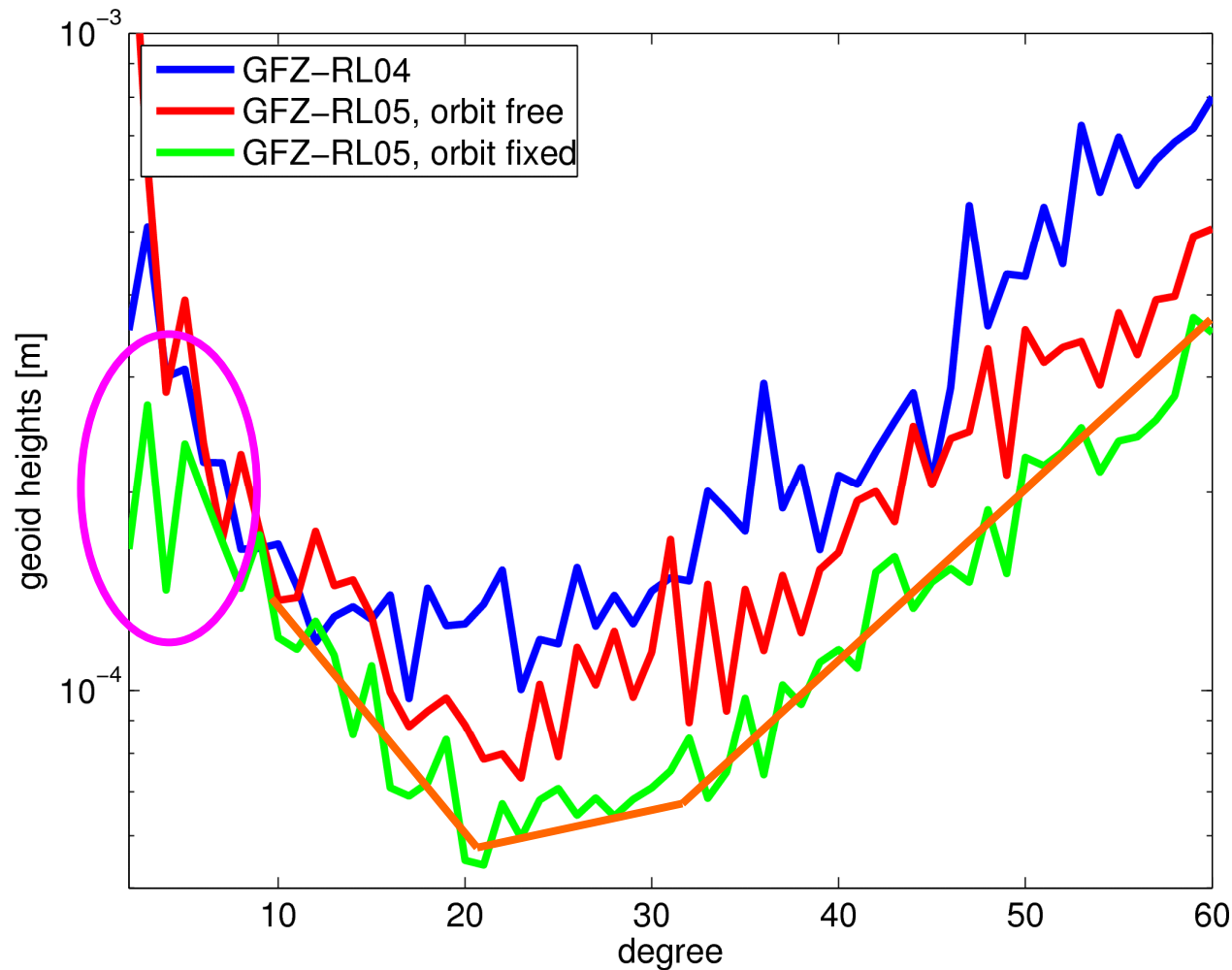
A priori model:
EIGEN-6C
(incl. time-var. d/o 50)

Stochastic accelerations:
60 min, in R,S,W

Signal



Signal and Noise in monthly models (GRACE)



Reference:
EIGEN-6C

Expected

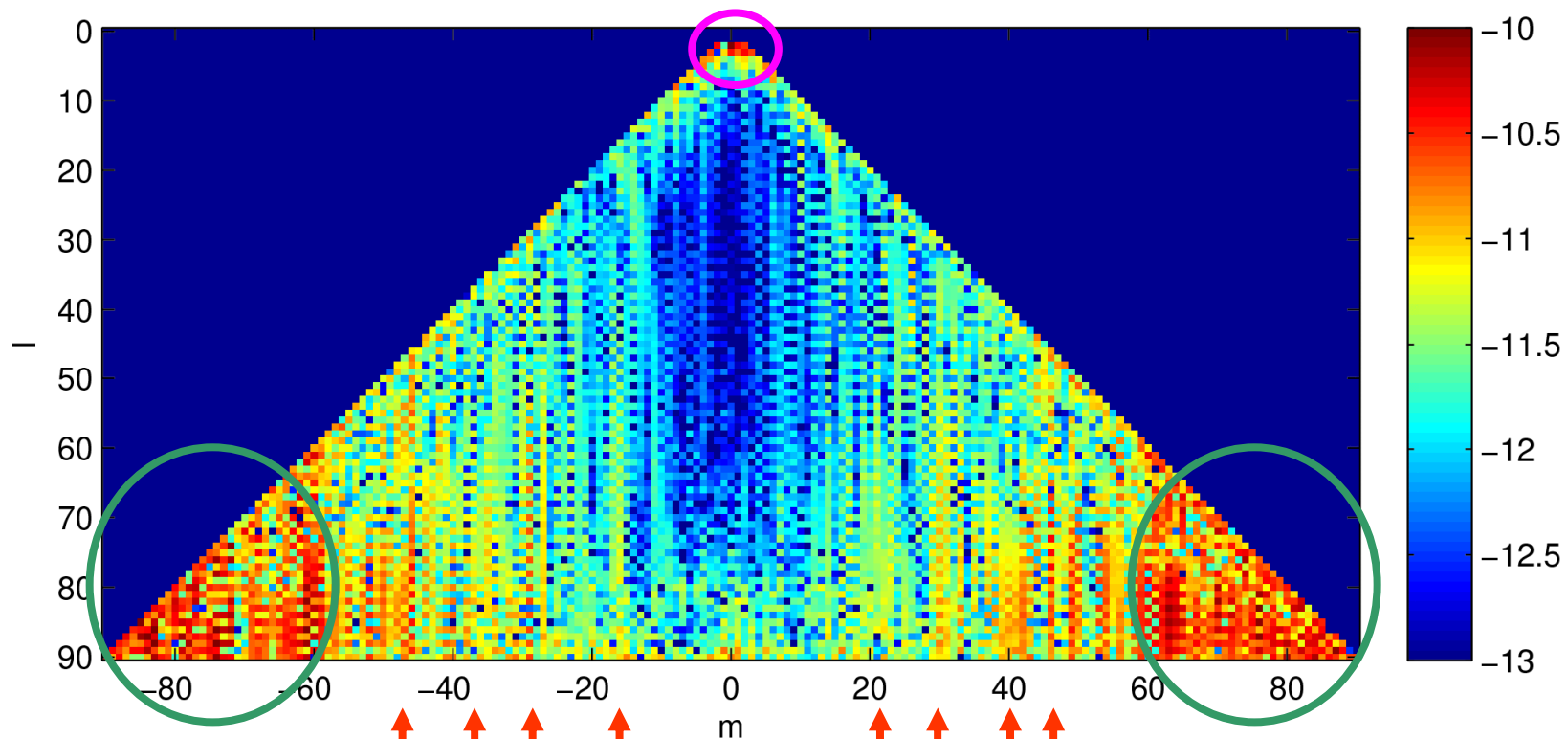
Surprising!

S,C-Coefficients

Difference: common estimation – orbit fixed

Example: March 2008

Expected



Direct – Spacewise – Timewise Analysis

- Direct approach:
generalized orbit determination problem
 - arc specific parameters
 - model parameters
- Space wise approach:
grid values are interpolated from observations
=> S,C–Analysis (integral formulas)
- Time wise approach:
observations as timeseries along orbit
Fourier–Analysis => Lumped Coefficients
=> Spherical Harmonic Coefficients

Timewise approach

- Potential along orbit
- Gravitational observations in satellite fixed frame
- Orbit perturbations relative to reference orbit (in satellite fixed frame)
- Inter-satellite observations
- Time derivatives

Gravity potential along orbit

$$\begin{aligned}
 V(r, I, u, \Lambda) = & \frac{GM}{r} \sum_{m=0}^L \sum_{k=-L}^L \sum_{l=\max(m, |k|)}^L \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I) \\
 & \left\{ \begin{array}{l} \bar{C}_{lm} \cos \psi_{km} + \bar{S}_{lm} \sin \psi_{km} \\ -\bar{S}_{lm} \cos \psi_{km} + \bar{C}_{lm} \sin \psi_{km} \end{array} \right\} \begin{array}{l} l-m \text{ even} \\ l-m \text{ odd} \end{array} \\
 = & \sum_{m=0}^L \sum_{k=-L}^L A_{mk}^V \cos \psi_{km} + B_{mk}^V \sin \psi_{km}
 \end{aligned}$$

Inclination Functions

Lumped Coefficients

$$\psi_{km} = ku + m\Lambda$$

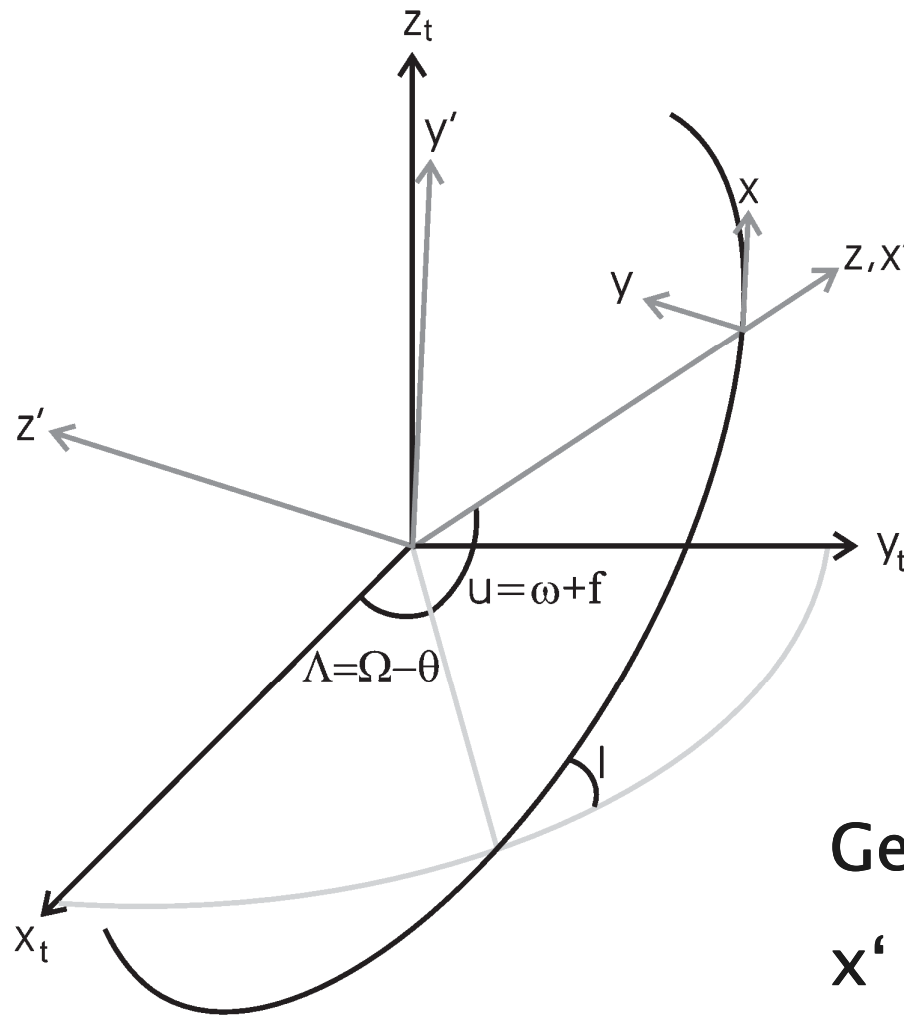
Lumped Coefficients: potential

$$A_{mk}^V = \sum_{l=\max(m,|k|)}^L \bar{H}_{lmk}^V \left\{ \begin{array}{l} \bar{C}_{lm} \\ -\bar{S}_{lm} \end{array} \right\}_{l-m \text{ even}}^{l-m \text{ odd}}$$

$$B_{mk}^V = \sum_{l=\max(m,|k|)}^L \bar{H}_{lmk}^V \left\{ \begin{array}{l} \bar{S}_{lm} \\ \bar{C}_{lm} \end{array} \right\}_{l-m \text{ even}}^{l-m \text{ odd}}$$

Transfer: $\bar{H}_{lmk}^V = \frac{GM}{r} \left(\frac{a_E}{r} \right)^l \bar{F}_{lmk}(I)$

Co-Rotating frame



Earth fixed frame:

x_t, y_t, z_t

Satellite fixed frame:

x = along-track

y = cross-track

z = radial

Geocentric, rotating frame:

$x' \parallel z, y' \parallel x, z' \parallel y$

Potential => gravitational acceleration

Gradient in
satellite
fixed frame:

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{1}{r \cos \phi'} \frac{\partial}{\partial \lambda'} \\ \frac{1}{r} \frac{\partial}{\partial \phi'} \\ \frac{\partial}{\partial r} \end{pmatrix}$$

$$\ddot{\underline{x}} = \nabla V$$



Transfer:

$$\bar{H}_{lmk}^x = \frac{1}{r} \frac{GM}{r} \left(\frac{a_E}{r} \right)^l \bar{F}_{lmk}(I) k$$

$$\bar{H}_{lmk}^y = \frac{1}{r} \frac{GM}{r} \left(\frac{a_E}{r} \right)^l \bar{F}_{lmk}^\times(I)$$

$$\bar{H}_{lmk}^z = -\frac{l+1}{r} \frac{GM}{r} \left(\frac{a_E}{r} \right)^l \bar{F}_{lmk}(I)$$

Gravitational accelerations => orbit perturbations

Hill (1878)

Equations
of motion:

$$\begin{aligned}\ddot{x} + 2n\dot{z} &= \frac{\partial \bar{T}}{\partial x} \\ \ddot{y} + n^2 y &= \frac{\partial \bar{T}}{\partial y} \\ \ddot{z} - 2n\dot{x} - 3n^2 z &= \frac{\partial \bar{T}}{\partial z}\end{aligned}$$

Perturbing potential

x, y, z relative to circular reference orbit (n const.)

Solvable analytically (exact)!

But only valid for circular orbits (approx.)

Transfer: orbit perturbations

$$\bar{H}_{lmk}^{dx} = \frac{(3n^2 + \dot{\psi}_{mk}^2)k - 2n\dot{\psi}_{mk}(l+1)}{\dot{\psi}_{mk}^2(n^2 - \dot{\psi}_{mk}^2)} \cdot \frac{GM}{a_E^2} \left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}(I)$$

Transfer: $\bar{H}_{lmk}^{dy} = \frac{1}{n^2 - \dot{\psi}_{mk}^2} \frac{GM}{a_E^2} \left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}^{\times}(I)$

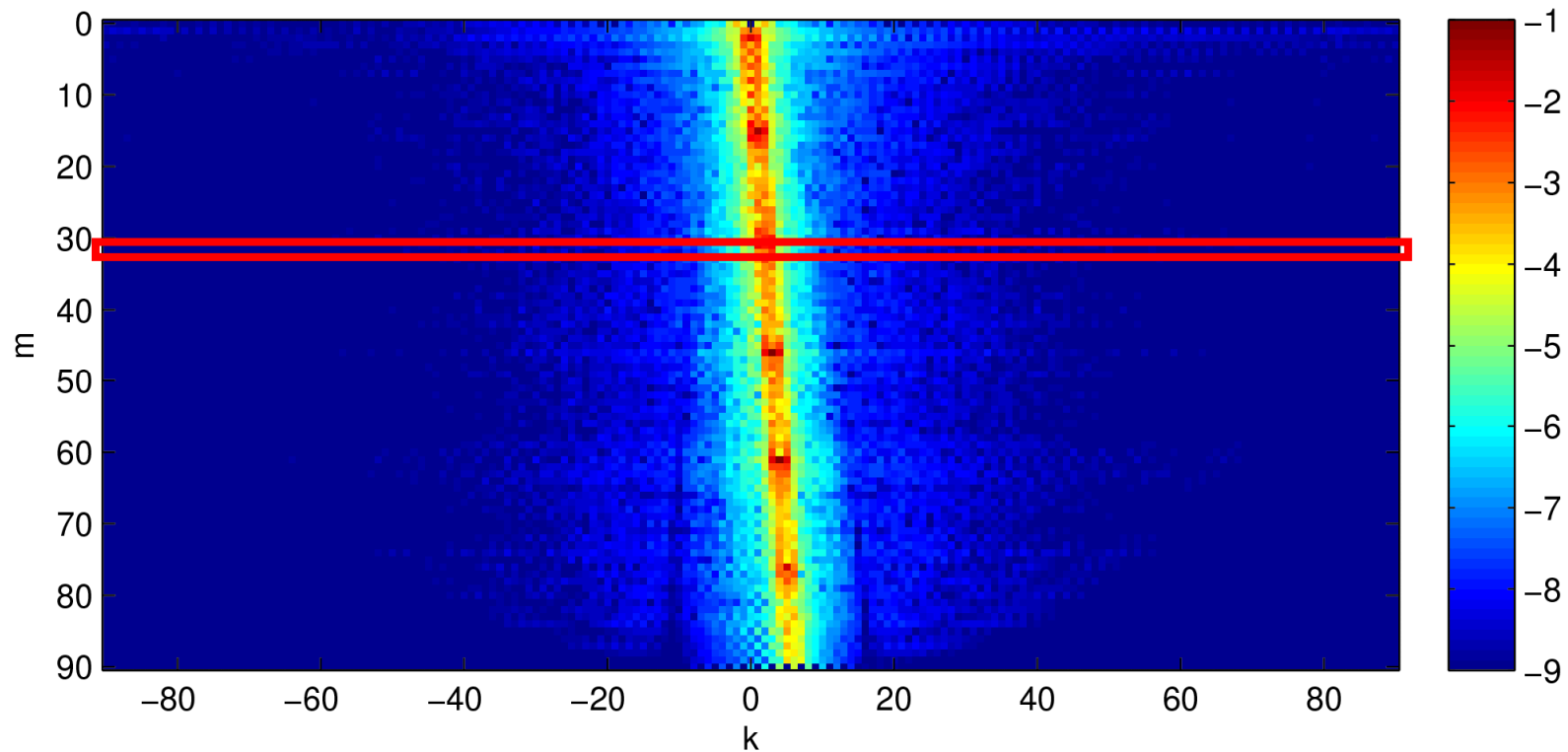
$$\bar{H}_{lmk}^{dz} = \frac{2nk - (l+1)\dot{\psi}_{mk}}{\dot{\psi}_{mk}(n^2 - \dot{\psi}_{mk}^2)} \cdot \frac{GM}{a_E^2} \left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}(I)$$

0-RESONANCE

N-RESONANCE

Lumped Coef.: Along-track orbit perturbations

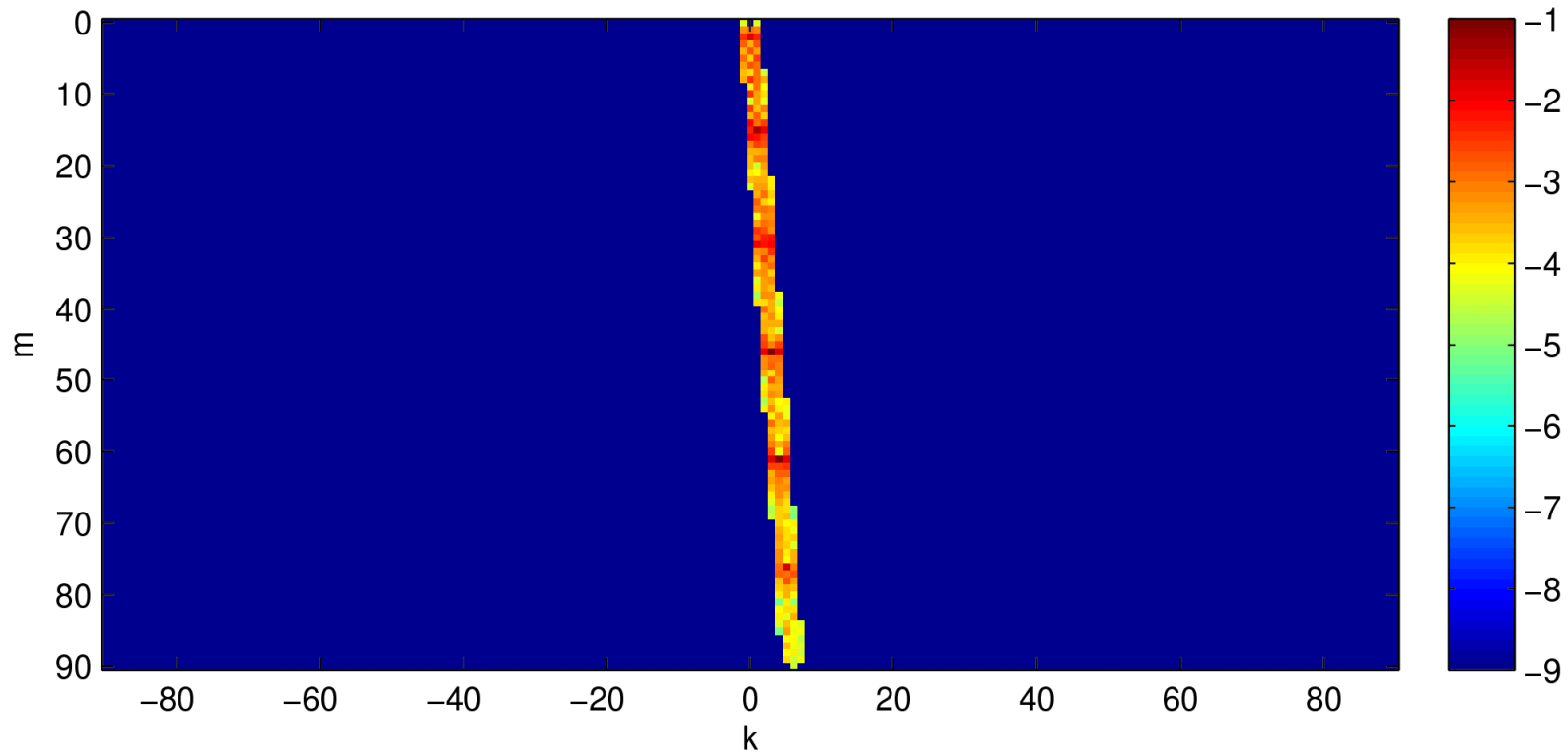
Difference: common estimation – orbit fixed



S_{lm} , C_{lm} depend on A_{mk} , B_{mk} of same order m

Lumped Coef.: Along-track orbit perturbations

Frequency < 24 rev/day

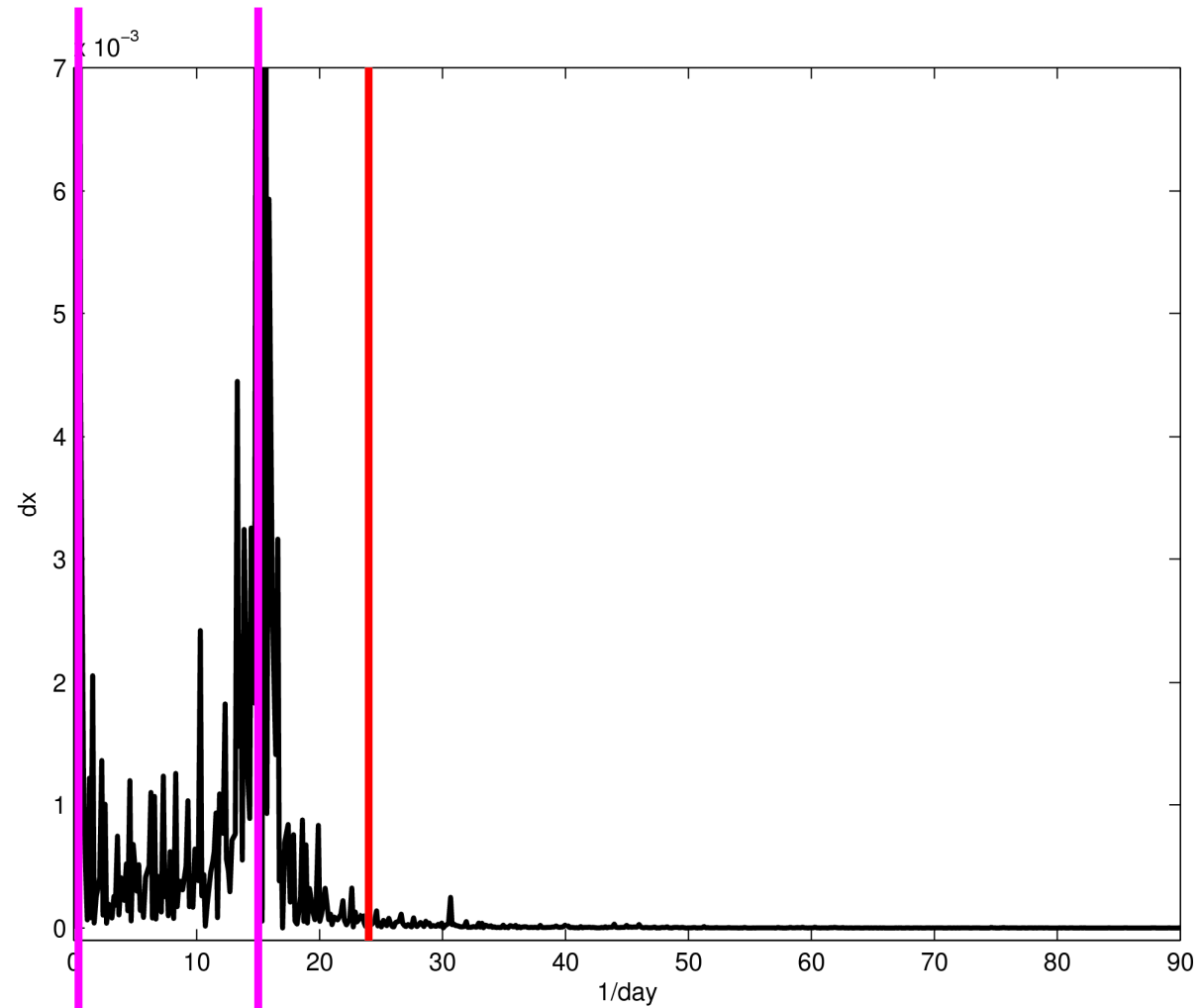


$$\dot{\psi}_{km} = k\dot{u} + m\dot{\Lambda}$$

Amplitude Spectrum (Lumped Coef. dx)

Frequency of
stoch.
accelerations

Resonance
of Transfer



Discussion

- Stochastic orbit parameters increase consistency between a priori and estimated gravity field.
- Aggravated when correlations are broken.
- Whole S,C-spectrum is affected by only few low frequent stochastic accelerations.
- Can be explained via lumped coefficients by timewise analysis.
- Could probably be useful to regularize lumped coefficients (instead of S, C).
- Is complicated by resonance effects in case of orbit perturbations and derivatives.